ROBUST IMAGE RESTORATION FOR ROTARY MOTION DEGRADATIONS AND THE MOTION PARAMETER IDENTIFICATION

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ABSTRACT

This paper constructs a frequency domain framework to deblur rotary degradation. The proposed method quantifies the rotary blur mechanism as frequency response and yields mean-square optimal estimation for original pixels under different noise levels. Then we present two methods to identify motion parameters. To detect correct rotary center, we propose a novel interval-searching method that works on the degraded pixel spectrum. The method is robust to noise. And a divide-conquer method is presented to iteratively calibrate the blur angle. Experimental results illustrate the frequency method outperforms spatial algorithms by up to 0.5 ~ 4 dB in Peak-Signal-Noise-Ratio (PSNR) and prove the methods for parameter identifications effective.

Index Terms—Frequency response, rotary motion blur, Wiener filter, divide-conquer algorithm, power spectrum

1. INTRODUCTION

Image degradations due to rotary motion are engendered by slow shutter speed relative to rapid rotation of the camera. Generally, not only may certain amount of additive noise contribute to the degradation, but also rotary parameters including the rotary center and the blur angle (the rotary angle spanned within the camera exposure time) are not precisely available. Hence, deblurring algorithms are required of strong tolerance with noise and are supposed to be able to recover motion parameters.

The main challenge is that the spatially variant property [1] of the rotary motion: the degradation gradually aggravates as a pixel leaves away from the rotary center. Noting that the rotary blur becomes spatially invariant among pixels with equivalent radius, a coordinate transform method [2] and the Bresenham circle based method [3] are proposed to deblur degraded images on a circle basis. These methods transplant the deconvolution method exclusive for the spatially invariant blur into this field. Unfortunately, the rotary blur kernel matrix is always singular, which is caused by some particular eigenvalues. However, in the regularized methods [2] and [3], the regularization factor exerts equivalent effects on all the eigenvalues and thus their solutions are over-regularized. In essence, their analyses of the rotary blur mechanism are confined within the spatial domain and do not explicitly protect eigenvalues that are well-conditioned. Another challenge is the blind restoration when motion parameters are not precisely available. [3] calculates the cross correlation value between the degraded pixel spectrum and possible blur kernels to identify the rotary parameters. However, the correlation value with respect to the correct blur kernel is not so distinguishing and sensitive to original pixels, which vary among different images.

In this work, we construct a frequency domain framework to deblur rotary motion degradation and identify the two motion parameters. Admittedly, deconvolution and frequency inverse filtering are essentially the same. However, in the frequency domain, ill-conditioned frequencies are explicitly calculable and

easy to manipulate. We filter them away but achieve optimal estimation for other frequencies at mean square sense, thereby avoiding the over-regularization in those well frequencies. As for motion parameter identification, we proceed with the degraded pixel spectrum analysis. If degraded pixels are fetched around the correct rotary center, their spectrum demonstrates uniformly distributed minima with constant intervals. Based on this fact, we present a zero-interval searching method to judge the rotary center. It is robust to noise. As for the blur angle, we introduce a divide-conquer method to search the blur length from on small error sets and ultimately yield the precise blur angle. Thus this method is computationally efficient. Experiment results reveal the proposed method outperforms spatial methods by 0.5 ~ 4 dB and also prove the parameter identification methods effective.

2. ROTARY BLUR AND WIENER FILTERING

Noting that the camera exposure time is short (less than 2ms), we assume the velocity of rotary motion remains constant within such period of time. Also, the principle blur originates from rotation as oppose to translation. So the rotary motion is regarded around a fixed center. Fig. 1 shows the relative motion between the camera and the scene: o is the rotary center and a (marked by rectangle) denotes a single pixel on the observed image plane. Many scenery points (marked by cross) pass through a during exposure time. With the assumption of constant velocity, the intensity of a degraded pixel equals to the mean luminosity of those scenery points. As all the passing scenery points align circularly with the degraded pixel, we disintegrate the degraded image into a group of circles around the rotary center and implement the deblurring circle by circle. In the context of digital image, the degradation equation within a circle is,

$$g(l) = 1/L \cdot \sum_{i=0}^{L-1} f(l-i) + n(l) \quad (1)$$

where $g(l)$ denotes the l th degraded pixel within the circle. $f(l-i)$ is the luminosity of the scenery point which locates i pixels away from $g(l)$. $n(l)$ denotes the Gaussian white noise incurred by the camera. L is the number of scenery points passing within the exposure duration. So L is proportional to three parameters, the radius r of the circle, camera angular velocity ω and exposure time T . Note that L not only denotes the discrete length of the arc where scenery points align but also reflects the amount of rotary blur, we term it as the “blur length”.

To obtain the blur function in frequency domain, we transform (1) using Discrete Fourier Transforms (DFT),

$$G(k) = H(k) \cdot F(k) + N(k) \quad (2)$$

where $H(k)$ is the frequency response of the rotary blur,

$$H(k) = \frac{1}{L} \sum_{i=0}^{L-1} e^{-j \frac{2\pi}{N} ik} = \begin{cases} 1 & k = 0 \\ \frac{1}{L} \cdot \frac{1 - \exp\{-j \frac{2\pi}{N} Lk\}}{1 - \exp\{-j \frac{2\pi}{N} k\}} & k = 1 \cdots N-1 \end{cases} \quad (3)$$

Note that $H(k)$ decays to zero when kL/N equals to an integer, the direct inverse filtering is ill-conditioned at the zeros of $H(k)$. Accordingly, we screen frequency zeros out and apply the Wiener filter into other frequencies. Thus, the optimal mean square estimation for original pixel spectrum is achieved.

$$\overline{F(k)} = G(k) \cdot \frac{H(k)^*}{|H(k)|^2 + S_n(k)/S_f(k)} \quad (4)$$

where S_n and S_f denote the spectrum density of the noise and original pixels, respectively. Since the two parameters are not precisely available, we substitute the ratio with the reciprocal of Signal Noise Ratio (SNR) to yield a parametric Wiener filter,

$$\overline{F(k)} = G(k) \cdot \frac{H(k)^*}{|H(k)|^2 + 1/\text{SNR}} \quad (5)$$

where $\text{SNR} = (S_g - S_n)/S_n$ and S_g is the degraded pixel spectrum,

$$S_g(k) = |H(k)|^2 \cdot S_f(k) + S_n(k) \quad (6)$$

Note that $H(k)$ is in the range from zero to one, $S_g - S_n$ can roughly approximate S_f . Therefore, the parametric Wiener with SNR is feasible with enough performance, which is illustrated by the sequent experiment.

Fig.1 Formation of the rotary blur

Fig.2 Original image cars

3. ROTARY MOTION PARAMETERS IDENTIFICATION

The rotary center and the blur angle are two important motion parameters to identify. The former guarantees pixels fetched circularly are subject to the blur equation (1); the latter relates to the blur length, which is a direct parameter of Wiener filter. In the context of digital image, the relationship between the blur length and the blur angle is nearly linear,

$$L = \lfloor N \cdot \theta / 360 \rfloor, \theta = \omega \cdot T \quad (7)$$

where $\lfloor \cdot \rfloor$ rounds a number to its nearest lower integer. N is the circumference and θ denotes the blur angle, which is the product of the rotary angle velocity ω and the exposure time T .

3.1. The Identification of the Rotary Center

Note that pixels on a circle around the rotary center correlates with each other in terms of a uniform rotary blur function, their spectrum should demonstrate regular patterns. On the other hand, for those circles fetched around other points, their pixels do not correlate and supposedly, no special patterns exist. The pattern is that in the degraded pixel spectrum, there is a series of minima uniformly distributing on the frequency axis with constant intervals. The intervals remain unchanged among spectrums in other circles. Degraded pixel spectrum S_g includes the blur term $|H(k)|^2 S_f$ and the noise term S_n . Note that some zeros distribute on $H(k)$ where $kL/N=1,2,\dots,L-1$, the blur term also contain uniform zeros in such frequency points k and their interval Δk equals to N/L . In addition, based on (7), N is proportional to L with the factor θ , which is constant among different circles. Therefore, all

circles in the degraded image share equivalent zero-intervals in their blur terms. In the existence of Gaussian white noise with a power spectrum roughly constant throughout all frequencies, the zeros in the blur term are increased by equivalent amount to be the minima in degraded pixel spectrum. Since these minima originate from zeros in H , we still name them as “zeros” and the intervals as “zero-interval” for convenience. In contrast, for circles fetched around other points, as the radius grows, the correlation of pixels drastically decline. Hence no obvious zero-interval exists in degraded pixel spectrum. Therefore, we may identify rotary center based on the existence of the constant zero-interval in the concentric circles.

A zero-interval searching method is introduced to evaluate the existence of zero-interval in a circle. The method is robust to noise as noise with constant power spectrum only affects the value of minima rather than their intervals. Nevertheless, the noise spectrum may fluctuate slightly all through the frequency axis. Indeed, the fluctuation may impair a particular minimum by making it not low enough to distinguish from its neighboring frequencies. This makes the direct searching for minima unreliable. However, since noise conforms to the Gaussian white noise model after all, the random fluctuation in the power spectrum should not be subject to an expectation of zero. In other words, the fluctuation may also sharpen the salience of other minima by enhancing their neighboring frequencies. Therefore, we use a candidate interval Δk to produce a series of frequency points and investigate their mean value. It attains the minimum value when the candidate interval equals to zero-interval. Meanwhile, $H(k)$ and the blur term equals to zero and only the noise term remains in (6). So the minimum of the mean value equals to the covariance of noise,

$$\Delta k_{\text{zero}} = \arg \min \left[1/M \cdot \sum_{i=1}^M S_g(i \cdot \Delta k) \right], M = \lfloor N/\Delta k \rfloor \quad (8)$$

$$1/M \cdot \sum_{i=1}^M S_g(i \cdot \Delta k) \geq \min(S_g) = S_n = \sigma^2 \quad (9)$$

We search each candidate zero-interval Δk and calculate the mean of their degraded pixel spectrum at the uniform frequencies produced by Δk . Based on (9), if the mean minimum is close to the noise covariance, we assert the existence of the zero-interval. Otherwise, we judge no valid zero-intervals exist on the circle. Then each circle votes about the existence of the zero-interval and the candidate with highest ballots is judged as the rotary center.

3.2 The Identification of the Blur angle

We propose a divide-conquer algorithm that iteratively rectifies the calculated blur length of each circle and ultimately yield a more precise blur angle. The divide step narrows down the identification of the blur length into one small set. The conquer step makes a fast and stable evaluation.

A rough blur angle can be recovered from the zero-interval, which is inversely proportional to the blur angle. If the blur angle suffers from some error, the resultant deviation in blur length L is amplified by the circumference N . Since L is represented by integers in the context of digital image, its deviation is denoted by integers as well. For inner circles with small circumferences N , the error of the blur angle merely engenders moderate deviation of the blur length. However, for outer circles, the error in blur angle is amplified by large N and then a large deviation of the blur length L is raised. With a rough blur angle and its error range, we let an error set (integer set) to denote the deviation of the calculated blur length from its correct value. Apparently, the error set of inner circles is small but that of outer circles are large. Noting that circumference increases gradually along neighboring

circles, the deviation accumulates slowly as oppose to an abrupt hopping. In other words, the error of blur length of an outer circle is enhanced comparing to its adjacent inner circle, but the enhancing amount is small.

We demarcate the degraded image into several adjacent ring belts (see Fig. 7), which are subject to three conditions,

- Classification condition: each belt is made of adjacent circles with the same error set, which is also the belt error set. For the circles in different belts, their sets are different.
- Adjacency condition for iteration: for adjacent belts, the error set of the outer belt is enlarged from the inner by merely two other elements, which are the maximum of the inner belt plus one and the minimum minus one.
- Initialization condition: the most inner belt has an error set of three elements, which are -1, 0 and 1, respectively.

Explicitly speaking, the i th belt have the error set as $\{-i, \dots, -1, 0, 1, \dots, i\}$. Given the rough blur angle θ_r and its error range $\Delta\theta_r$, we derive the circumferences N of circles on belt i are subject to,

$$i-1 < \lfloor N \cdot (\theta_r + \Delta\theta_r) / 360 \rfloor - \lfloor N \cdot (\theta_r - \Delta\theta_r) / 360 \rfloor \leq i$$

(10) which means the difference between the maximum possible blur length and the minimum in the i th belt should be not more than i . Admittedly, the constraint imposed by (10) is slightly looser as the maximum-minimum blur length difference is larger than the blur length error, thereby expanding the error set. However, the drawback is bypassed because the divide step narrows an error set to one of its subsets with two or three elements.

Adjacency condition guarantees the blur length error only increases by one among the adjacent belts. Supposes belt $i-1$ has error j and we decompose the error set of belt i $\{-i, \dots, -1, 0, 1, \dots, i\}$ into subsets $\{-i, -i-1\}, \dots, \{-2, -1\}, \{-1, 0, 1\}, \{1, 2\}, \dots, \{i, i+1\}$. In fact, only $\{j, j+1\}$ ($\{j-1, j\}$ when j is negative) needs to be investigated. Under the initialization condition, we judge the deviation of belt 1 from $\{-1, 0, 1\}$ and repeat the identification iteratively until the error of the last belt is rectified. Meanwhile, the blur angle is identified by dividing the calibrated blur length of the most outer circle with its circumference.

Accordingly, given a blur length L calculated from the rough blur angle, we only need judge the correct blur length from $L-1$, L or $L+1$. The conquer step discusses an inverse filter method to rule out two false blur lengths and thus yield the error of the circle. The frequency zeros of these inverse filters are removed in case of the abnormal amplification of noise. Note that the noise is outweighed by the power of blurred pixels at median frequency domain, the trivial effects of noise at median frequency domain is neglected. The inverse filtering result is,

$$\overline{F(k)} = \frac{H(k, L)}{H(k, L \pm 1)} \cdot F(k) = \frac{1 - \exp\{-j 2\pi / N \cdot L k\}}{1 - \exp\{-j 2\pi / N \cdot (L \pm 1) k\}} \cdot F(k) \quad (11)$$

$$1 < \left| \frac{1 - \exp\{-j 2\pi / N \cdot L k\}}{1 - \exp\{-j 2\pi / N \cdot (L \pm 1) k\}} \right| < \infty$$

$$k_{\max} = N / (L \pm 1) - 1, 2 \cdot N / (L \pm 1) - 1, \dots, n \cdot N / (L \pm 1) - 1$$

where $k_{\max}$ are frequencies that make the ratio of two blur functions maximum. At the median frequencies in $k_{\max}$, the ratio grows very large for its small denominator cannot be cancelled with the numerator. The large ratio results in the abnormal peaks at the corresponding frequencies of the deblurred pixel spectrum, which should be around zero. On the other hand, if we deblur with the correct blur length, the ratio is equal to one constantly. So there is no amplification for any frequencies. Therefore, the blur length is distinguished by investigating whether the deblurred pixel median spectrum at $k_{\max}$ are extremely high.

4. EXPERIMENTAL RESULTS AND CONCLUSIONS

4.1 The Deblurring with the Known Parameters

An aerial grayscale image *cars* (Fig. 2) is used as the test image for the deblurring with the parametric wiener filter. The degraded versions images are synthetic, in order to bypass problems which are beyond the scope of this paper, such as turbulent blur. We blur *cars* with the blur angle 15 deg and contaminate them with additive Gaussian noise with SNR of 40dB, 30dB and 20dB, respectively. With the blur angle and SNR, the parametric Wiener filter is developed based on (5) to deblur the images. Fig. 3 depicts the deblurring results.

Fig. 3 Deblurring results: Upper: degraded images, Left-Right: SNR: 40dB, 30dB, 20dB; Middle: deblurred by spatial method, Left-Right: PSNR(dB) 27.31, 27.16, 25.92; Bottom: deblurred by frequency method, Left-Right: PSNR(dB) 31.58, 27.67, 26.5

4.2 The Identification of the Rotary Center

The degraded image is rotated around the point (128,128) and with the SNR of 30dB. In practice, the rotary center is not precisely known (if known we don't have to identify it). We apply the zero-interval searching method to identify the rotary center from 25 candidate points and assess its performance by comparing the identified center with (128,128). We detail the operation in one circle at the radius of 60 and all the other circles follow the similar procedures. For candidate (128,128) and (128,129), according to (8), the spectrum means vs. different intervals are illustrated in Fig. 4.

To judge the existence of the zero-interval by the constraint in (9), a threshold is set to cut off the invalid candidate intervals. Here we take the threshold as 1.2 times of the noise covariance, which is 0.12. Note that the spectrum of (128,128) decreases to its valley when the interval equals to 38° and the minimum value is less than the threshold. In contrast, the spectrum mean of (128,129) doesn't have any valley value less than the threshold. Therefore, the circle votes for (128,128) but against (128,129). Each circle gives a vote for a rotary center candidate. Fig. 12

¹ The mean value with respect to the multiple of the zero-interval may be even less, so the algorithm is implemented as: find the points below the threshold and the minimum point, remove the points that are not the divisor of the minimum point. Then assign the zero-interval as the minimum divisor.

illustrates the statistics of final ballots for 25 center candidates under different noise levels. In all noise levels, the candidate (128,128) outnumbers other candidates to be the rotary center.

Fig. 4 The spectrum mean vs. intervals

Fig. 5 The ballots of rotary center candidates

Fig. 6 The median peaks in the deblurred pixel spectrum

Fig. 7 Left: the demarcations of 10 belts;
Right: robust deblurring with imprecise blur angle 10 deg

4.3 The Identification of the Blur Angle

The test image *cars* is blurred with 12 deg but we suppose the calculated blur angle from the zero-interval is 10 deg along with the error range $-5 \text{ deg} \sim 5 \text{ deg}$. We also detail the operation on a circle on belt 1 and focus on the error transition to circles on the next belt. Based on (10), the circumference N of a circle on belt i is from $72(i-1)$ to $72i$. For the circle with the radius of 13 ($N=68$),

the calculated blur length is $\text{int}(10/360 \times 68) + 1 = 2$. With the initial error set $\{-1, 0, 1\}$, the candidate blur length are $\{1, 2, 3\}$. Using inverse filter (after removal of zeros) with the three blur lengths, the deblurred pixel spectrums are illustrated in Fig. 6.

Since the spectrums with the candidate blur length 1 and 2 have median peaks, they should be ruled out. For the other circles on the same belt, we adopt a voting strategy: each circle votes for a candidate blur length and we judge the one with the highest ballot as the correct blur length. Here, belt 1 evaluates the blur length as 3 and the corresponding error as 1. Then the error set for the belt 2 is $\{1, 2\}$. Similarly, for the circle at a radius of 20 ($N=104$) in belt 2, the calculated blur length is $\text{int}(10/360 \times 104) + 1 = 3$. Hence the candidate blur length are $\{4, 5\}$. Similar operation is done on this circle and the candidate blur length 5 is ruled out. We rectify the circles iteratively and the resultant rectified blur length chain for each belt is 3, 5, 8, 10, 12, 15, 17, 20, 22 and 24. With the last circle circumference 716, we rectify the blur angle as $360/716 \times 24 = 12.06 \text{ deg}$. Therefore, the blur angle is effectively recognized. Fig. 7 shows the deblurring results with the imprecise blur angle 10 deg.

We propose a frequency framework to deblur rotary motion degraded images. This method avoids the over-regularization of previous spatial algorithms. We use the constant zero-interval in the degraded pixel spectrum to identify the rotary center and present a zero-interval searching method, which is robust to noise. Then, a divide-conquer algorithm is introduced to search the blur length from small error sets. It rectifies the blur length iteratively and finally yields precise blur angle. Experiment results indicate the proposed method effectively deblurs the degradations under different noise levels and prove the identifications of blur parameters are effective.

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